

Twisted-pair transmission-line distributed parameters

For more accurate modeling of twisted-pair transmission-line distributed parameters, you must take into account several important factors. Material and structural make-ups, twist pitch, skin effect, and proximity effect all play a role.

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Abstract

Both unshielded-twisted-pair (UTP) and shielded-twisted-pair (STP) transmission lines are widely used from your office's Ethernet to safety-critical avionics applications. Despite universal usage of both types of transmission lines, references to accurate calculation of the twisted-pair parameters are mostly missing, especially for high-frequency applications. Papers on transmission lines often address microstrip or coax cable, and anyone can readily find the equations for these types, but you can rarely find practical guidance on twisted pair. For example, most textbooks model the twisted pair using the parallel wire configuration and cannot explain the change in bus impedance (hence, reflection) when the wires are separated for magnetic coupling (current-mode transceiver). In this article, we analyze the parameters that reflect more accurately the physical twisted-pair geometry. These parameters take into account not only the measured dimension, structural make-up, and material constants but also the pitch of twist, the proximity effect of current-carrying wires, the dielectric dispersion, and the effects of frequency on skin depth resistance.

Introduction

Most transmission-line textbooks can provide the derivation of bus impedance for simple geometry, such as parallel wire using the Cartesian coordinates or coaxial cable using the polar coordinates. Most analyses model the twisted pair after the parallel wire and replace the air environment with a uniform dielectric. These models are unsuitable particularly at high frequency or for applications in which the signal integrity at every point along the bus is of high

importance. Using MathCAD, we will explore the frequency-dependent behaviors of the many so-called "constants" of a lossy transmission line.

Concerning simulation, most standard SPICE models don't allow for frequency-dependent terms that are necessary for accurate modeling. With more accurate calculation of the distributed parameters, you can model the attenuation and dispersion in the transmission line as a function of frequency with high fidelity. You can compute the line response using the distributed model rather than the lumped ladder model. Circuit modeling for the twisted-pair transmission line will be the topic of an upcoming article.

Transmission-line parameters

The characteristic impedance, Z_0 , of a transmission line is a complex quantity. It is a function of the distributed parameters (per unit length) series resistance R , series inductance L , shunt capacitance C , and shunt conductance G . These parameters are modeled after the geometry and material make-up of the transmission line (wave propagation, energy storage, conductor loss, dielectric loss, and radiation loss).

$$Z_0 = \sqrt{(R + j\omega L) / (G + j\omega C)} = R_0 + jX_0$$

With both resistance and reactance specified in ohms, a common misconception for a given $Z_0 = 120\Omega$, for example, is that the impedance is purely resistive and independent of frequency. This situation is true only for a lossless transmission line in which $R=G=0$ or for a distortionless transmission line in which $R/L=G/C$ when Z_0 becomes $\sqrt{L/C}$, which is real.

In general, we have a lossy twisted-pair transmission line for which the parameters R, L, C, and G are nonzero and are themselves functions of frequency. What we need is the characterization of a bus cable that shows the variation of its magnitude and phase with frequency. It is only if the impedance remains fairly constant over our frequency range of interest that we can treat the characteristic impedance as a constant. We then can specify the load impedance to match the transmission line for that particular range of frequency.

UTP impedance calculation

Below are the constants and specification for a given 120Ω UTP used in this analysis (all units in mks).

Wire length:	$l := 50$
Wire Spacing:	$D := 2.11 \cdot 10^{-3}$
Conductor Diameter:	$d := 1.12 \cdot 10^{-3}$
Virgin PTFE dielectric constant:	$\epsilon_r := 2.1$
Measured effective UTP permittivity	$\epsilon_{tp} := 1.56$
Air permittivity:	$\epsilon_0 := 8.854 \cdot 10^{-12}$
Dielectric permiability:	$\mu := 4 \cdot \pi \cdot 10^{-7}$
Copper conductivity:	$\sigma_1 := 5.8 \cdot 10^7$
Silver (plating) conductivity:	$\sigma_2 := 6.17 \cdot 10^7$
Frequency:	$f := 10^3, 10^4 \dots 10^7$
Nominal frequency:	$f_0 := 6 \cdot 10^6$
Dielectric loss tangent:	$P_f := 2 \cdot 10^{-4}$
Number of twists (/m):	$T := 20$

All equations, calculations, and plotting in this analysis use MathCAD software, which is a powerful and easy-to-use math worksheet. After establishing the equations, you can substitute the transmission-line values with your own twisted-pair parameters and watch all the calculated results and plots change instantly.

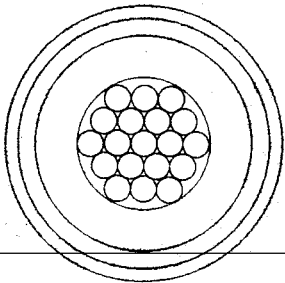
In general, the permittivity of a dielectric is a complex number and changes with frequency: $\epsilon = \epsilon_r \epsilon_0 = \epsilon' + j\epsilon''$. However, between 1 and 100 MHz, the permittivity of PTFE (polytetrafluorethylen, or Teflon) is almost constant at 2.1, whereas its dissipation factor is 0.0002 (Reference 1).

The same is true for the permeability of the conductor, which is a complex number and changes with frequency: $\mu = \mu_r \mu_0 = \mu' + j\mu''$.

However, μ_r can change drastically at the low (kilohertz) level of frequency. At the megahertz level, the relative permeability of both copper (0.999991) and silver (0.999998) is practically unity, and thus we will use μ_0 in the equations (Reference 1).

If you use the virgin dielectric constant, $\epsilon_r=2.1$, for PTFE, you would miscalculate the impedance using the simple parallel wire-over-ground-plane geometry that most textbooks list.

In this example, the conductor for the twisted pair consists of 19 silver-plated strands of copper and multiple layers of dielectric with different material constants as the figure below illustrates:



The calculation for the helical pitch angle of twist (Reference 2) plus the bus length (before twisting) and velocity of propagation are:

$$\theta := \text{atan}(T \cdot \pi \cdot D)$$

$$\theta = 7.55 \cdot \text{deg}$$

$$l_0 := T \cdot l \cdot \pi \cdot D \cdot \sqrt{1 + \frac{1}{\tan(\theta)^2}}$$

$$l_0 = 50.44$$

$$v_p := \frac{1}{\sqrt{e_{tp}}}$$

$$v_p = 0.8$$

Using the twist angle as a factor to compensate for the equivalent dielectric constant and with q as the twist correction factor, the expression for the equivalent dielectric constant, $e_{eq} = 1 + q(e_r - 1)$, is based on the facts that $e_{eq} = 1$ for all air ($e_r = 1$) and $e_{eq} = e_r$ for all dielectric ($q = 1$). The parallel equivalent of these layers should match the measured overall dielectric constant of 1.56.

We can calculate the impedance of the twisted pair (still assuming losslessness) as follows:

$$q := 0.45 + 1 \cdot 10^{-3} \cdot \left(\theta \cdot \frac{180}{\pi}\right)^2$$

$$q = 0.51$$

$$e_{eq} := 1 + q \cdot (e_r - 1)$$

$$e_{eq} = 1.56$$

$$C_{eq} := \frac{(\pi \cdot \epsilon_0 \cdot e_{eq})}{\text{acosh}\left(\frac{D}{d}\right)}$$

$$C_{eq} = 3.47 \cdot 10^{-11}$$

$$L_{eq} := \frac{(\mu \cdot \epsilon_0 \cdot e_{eq})}{C_{eq}}$$

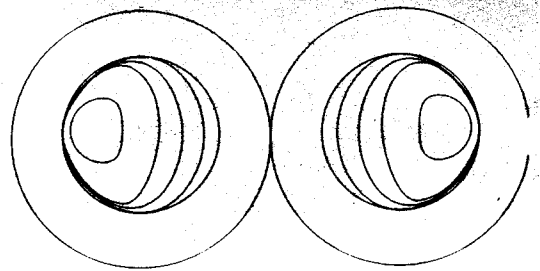
$$L_{eq} = 4.99 \cdot 10^{-7}$$

$$Z_{eq} := \sqrt{\frac{L_{eq}}{C_{eq}}}$$

$$Z_{eq} = 119.83$$

Unlike coaxial cable, the twisted-pair bus has two adjacent conductors that give rise to the proximity effect in which the current distribution is not uniform as the next figure illustrates.

Lumping all of the wire strands together, the proximity correction factor for two wires that are right next to each other is $P = 1.15$ (Reference 3).



As the bus-core figure illustrates, the diameter of each strand is about one-fifth of the overall conductor diameter. The cross-sectional area of each strand is therefore $1/25$ the overall conductor area.

For this lossy twisted-pair line, you calculate the skin depth and dc resistance as follows (taking into account the proximity factor and the conductor area ratio as a good approximation):

$$\delta(f) := \frac{1}{\sqrt{\pi \times f \cdot \mu \times \sigma_2}}$$

$$R_{dc} := (1.15) \cdot \left(\frac{19}{25}\right) \cdot \frac{2}{\left[\pi \cdot \left(\frac{d}{2}\right)^2 \cdot \sigma_2\right]}$$

$$R_{dc} = 0.029$$

At high frequency, the skin effect forces most of the propagation of electrical signal to the silver plating. You commonly use silver for better conductivity and better corrosion resistance.

The setup for the equivalent dielectric constant as a function of the twist pitch is as follows:

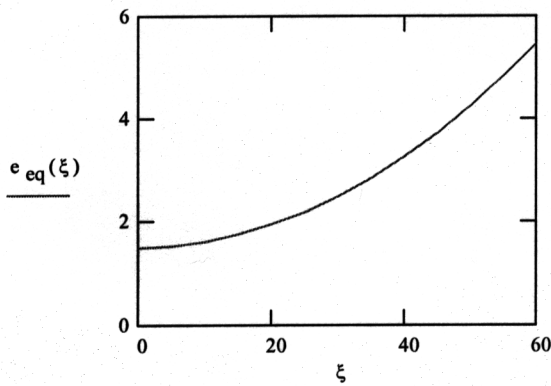
$$\xi := 0.5 \dots 60$$

$$e_r := 2.1$$

$$q(\xi) := 0.45 + 1 \cdot 10^{-3} \cdot (\xi)^2$$

$$e_{eq}(\xi) := 1 + q(\xi) \cdot (e_r - 1)$$

This setup results in the following plot:

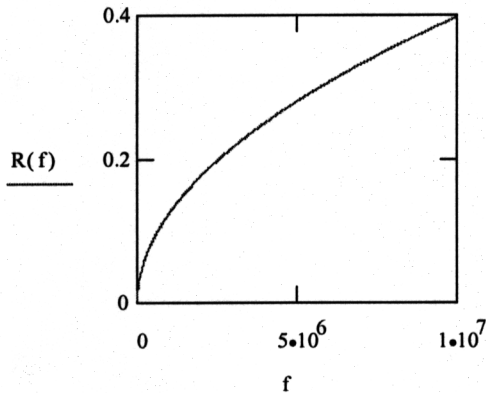


The distributed ac resistance (/m) is then:

$$R(f) := R_{dc} \cdot \frac{d}{4 \cdot \delta(f)}$$

$$R(f) := R_{dc} \cdot \frac{d}{4} \cdot \sqrt{\pi \cdot f \cdot \mu \cdot \sigma_2}$$

which results in the following plot of ac resistance versus frequency:



And the distributed shunt conductance is:

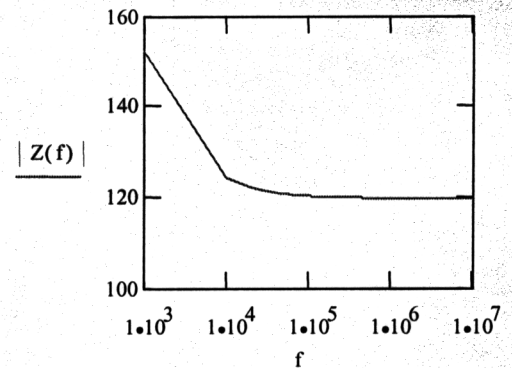
$$G(f) := 2 \cdot \pi \cdot P_f \cdot C_{eq}$$

You can then calculate the lossy characteristic impedance and plot its magnitude and phase in the following figures. You can see that for frequencies

higher than 1 MHz, the impedance is approximately constant at 120Ω and is almost purely resistive with a small phase angle.

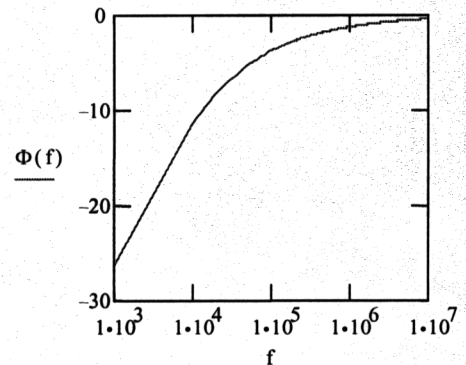
The following is the magnitude of Z(f):

$$Z(f) := \sqrt{\frac{\left(\frac{1}{4} \cdot R_{dc} \cdot d \cdot \sqrt{\pi \cdot f \cdot \mu \cdot \sigma_2} + j \cdot 2 \cdot \pi \cdot f \cdot L_{eq}\right)}{\left(2 \cdot \pi \cdot f \cdot P_f \cdot C_{eq} + j \cdot 2 \cdot \pi \cdot f \cdot C_{eq}\right)}}$$



The following is the phase angle of Z(f):

$$\Phi(f) := \text{angle}(\text{Re}(Z(f)), \text{Im}(Z(f))) \cdot \frac{180}{\pi} - 360$$



The propagation constant, $\gamma = \alpha + j\beta$ (in which α is the attenuation constant in neper/m and β is the phase constant in radian/m) is not really a constant. It is dependent on frequency in a complicated way:

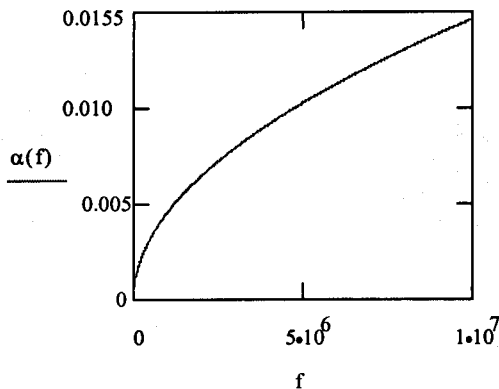
$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L) \cdot (G + j\omega C)}.$$

The MathCAD expression for the propagation constant is:

$$\gamma(f) := \sqrt{\left(\frac{1}{4} \cdot R_{dc} \cdot d \cdot \sqrt{\pi \cdot f \cdot \mu \cdot \sigma_2} + j \cdot 2 \cdot \pi \cdot f \cdot L_{eq}\right) \cdot \sqrt{\left((2 \cdot \pi \cdot f \cdot P_f \cdot C_{eq} + j \cdot 2 \cdot \pi \cdot f \cdot C_{eq})\right)}}$$

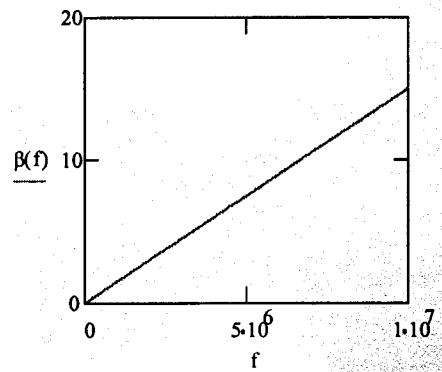
Depending on whether the cases are lossless, low loss, or distortionless, the attenuation and phase factors can be approximately constant or linear with frequency. The phase velocity, or velocity of propagation ($v_p = \omega/\beta$), is constant only if β is approximately a linear function of frequency. The following plot shows the attenuation in decibels/meter:

$$\alpha(f) := \text{Re}(\gamma(f)) \cdot \left(\frac{20}{\ln(10)}\right)$$



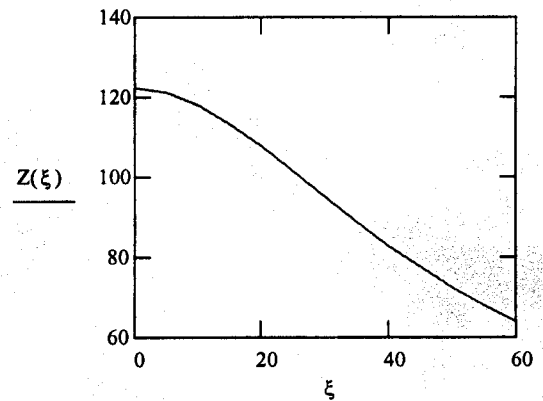
The phase shift in degrees/meter is:

$$\beta(f) := \text{Im}(\gamma(f)) \cdot \frac{180}{\pi}$$



You can calculate the effect of the twist angle on the impedance as follows:

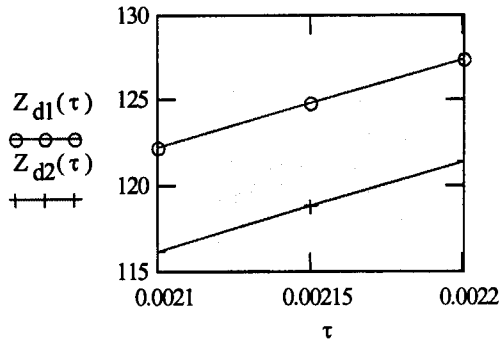
$$Z(\xi) := \frac{\text{acosh}\left(\frac{D}{d}\right)}{\pi} \cdot \sqrt{\frac{\mu}{\epsilon_0 \cdot \epsilon_{eq}(\xi)}}$$



The impedance of the twisted pair is fairly constant with a fixed twist angle. Nonuniformity in conductor diameters and dielectric thickness, together with variation in the twist angle, can significantly affect the bus impedance. Intuitively, you can see why the impedance increases when the twisted wires are separated for current-mode transceiver coupling: As the wires are separated to form an eye, the capacitance decreases, and the inductance increases.

The following plot shows the impedance variance with 0.001-in. manufacturing tolerances on the

conductor diameter, d , and the overall dielectric diameter, τ :



With this plot in mind, you can see that the bus terminator need not exactly match the specified bus impedance because the impedance likely varies by a small percent. Because the resistive termination swamps any small presence of reactive load, it has to match only to a nominal bus-impedance value in the middle of the above impedance band. Even at 5% mismatch, the reflection coefficient would be $(126-120)/(126+120)=2.4\%$, which results in reflected power of only 0.12%.

Armed with this information, you can first analyze the input impedance as a function of frequency at a bus length of 50m and a 5% mismatched load at 126Ω .

$$l := 50$$

$$Z_L := 126$$

$$f := 1 \cdot 10^6, 1.1 \cdot 10^6 \dots 10 \cdot 10^6$$

$$R(f) := 0.06 + 4.69 \cdot 10^{-4} \cdot \sqrt{f}$$

$$G(f) := 6.17 \cdot 10^{-12} \cdot f$$

$$L := 4.99 \cdot 10^{-7}$$

$$C := 3.47 \cdot 10^{-11}$$

Using the theoretical propagation constant, calculate the characteristic impedance and input impedance as functions of frequency:

$$\gamma_2(f) := \sqrt{(R(f) + j \cdot 2 \cdot \pi \cdot f \cdot L) \cdot (G(f) + j \cdot 2 \cdot \pi \cdot f \cdot C)}$$

$$Z_o(f) := \sqrt{\frac{R(f) + j \cdot 2 \cdot \pi \cdot f \cdot L}{G(f) + j \cdot 2 \cdot \pi \cdot f \cdot C}}$$

$$\alpha_2(f) := \operatorname{Re}(\gamma_2(f)) \cdot \left(\frac{20}{\ln(10)} \right)$$

$$Z_{in2}(f) := Z_o(f) \cdot \left(\frac{Z_L \cdot \cosh(\gamma_2(f) \cdot l) + Z_o(f) \cdot \sinh(\gamma_2(f) \cdot l)}{Z_o(f) \cdot \cosh(\gamma_2(f) \cdot l) + Z_L \cdot \sinh(\gamma_2(f) \cdot l)} \right)$$

Although MathCAD can calculate all the complex math functions, it is useful to find an easy way to calculate some real numbers for simple comparison. You can calculate the bus impedance, Z , and propagation constant, γ , for a particular frequency from the distributed RLCG parameters as follows:

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{(j\omega L)(j\omega C) \left(1 + \frac{R}{j\omega L}\right) \left(1 + \frac{G}{j\omega C}\right)} \end{aligned}$$

For high-quality, low-loss cable at high frequencies (more than a few megahertz), in which $\omega L \gg R$ and $\omega C \gg G$, you can use the following simplification for the characteristic impedance:

$$Z = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}$$

Also, by using the approximation

$$(1+a)(1+b) \approx (1+a+b) \text{ and } \sqrt{1+c} \approx 1 + c/2$$

for $a, b, c \ll 1$, you can manually evaluate the propagation constant as follows:

$$\gamma = \left(\frac{R}{2\sqrt{L/C}} + \frac{G\sqrt{L/C}}{2} \right) + j\omega\sqrt{LC}$$

In this equation, the first term gives the attenuation, and the second term gives the phase shift (per unit length). You can calculate the time delay per unit length from $\sqrt{LC}$. Note that although the distributed RLGC parameters are themselves theoretically functions of frequency, the distributed L and C parameters are mostly constant at high frequency for high-quality cable; manufacturers often supply these parameters.

The high-frequency MathCAD expressions for the propagation constant, characteristic impedance, and input impedance are therefore:

$$\gamma(f) := \frac{R(f)}{2 \cdot \sqrt{\frac{L}{C}}} + \frac{G(f)}{2} \cdot \sqrt{\frac{L}{C}} + j \cdot 2 \cdot \pi \cdot f \cdot \sqrt{L \cdot C}$$

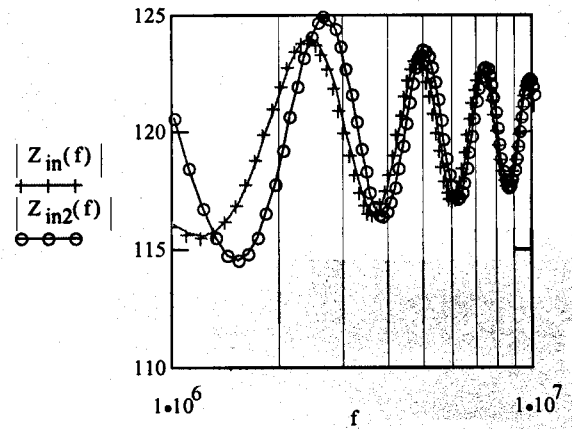
$$Z_0 := \sqrt{\frac{L}{C}}$$

$$Z_{in}(f) := Z_0 \cdot \left(\frac{Z_L \cdot \cosh(\gamma(f) \cdot l) + Z_0 \cdot \sinh(\gamma(f) \cdot l)}{Z_0 \cdot \cosh(\gamma(f) \cdot l) + Z_L \cdot \sinh(\gamma(f) \cdot l)} \right)$$

$$\alpha(f) := \text{Re}(\gamma(f)) \cdot \left(\frac{20}{\ln(10)} \right)$$

The following figure plots the input impedance with a 126Ω mismatched load to amplify the standing-wave effect. Note that you can see a similar result if you plot the input impedance as a function of bus length at a specific frequency, say 5 MHz, with the same mismatched load.

At frequencies greater than 5 MHz, you can see that the high-frequency approximation closely tracks the theoretical input impedance:



The beauty of the characteristic impedance, Z , of a transmission line is that although a high-quality, low-loss line comprises mostly inductive and capacitive reactance, its characteristic impedance is resistive at high frequency; that is, the line looks like a simple resistor. This situation is true only when the line is infinitely long or when the line terminates with a matching resistor. When a signal propagates down a transmission line with matching termination, no reflection occurs just as is the case when the line is infinitely long.

When a line is finite in length and ends with a mismatched load, its characteristic impedance changes with frequency and also varies along the length of the line. The problem here is the formation of standing waves on the line due to the interference of reflected and incident signals. Constructive interference would result in impedance peaks, but destructive interference results in impedance nulls. This situation occurs because the characteristic impedance of a line is the voltage-to-current ratio of a traveling wave when propagating down the line.

One example of an extreme change in impedance with frequency is when the frequency coincides with an odd multiple of a quarter wavelength. Here a short-circuit load would look like an open circuit (and vice versa) when viewed at the source ($Z_{in} = Z_0^2 / Z_L$ for a quarter-wave transformer).

STP bus

As for STP, the shield now contains the electric field, and this distribution represents additional shunt capacitance from each twisted wire to the shield. For the differential mode of transmission, the equivalent effect is increased distributed capacitance, C ; decreased impedance; and longer propagation delay. The extra dielectric insulation between the shield and the already-insulated UTP also increases the distributed conductance, G , which represents dielectric loss.

Note that the shield has no effect on the magnetic field. With differential mode of propagation, the shield carries no net current (except for common-mode noise current that results from the transmission-line imbalance).

The parameters for STP are therefore similar to those of UTP, except for the increased capacitance and conductance. The dc resistance, R_{dc} , and skin-depth resistance, R_{ac} , also change due to the proximity effect (change in surface current distribution) from the shield.

Summary

Using MathCAD, this article has constructed the frequency-dependent equations of the many so-called constants of a lossy transmission line. You need to be careful to find out all of the assumptions; be careful not to take a given component constant at its surface value. Manufacturers' specifications usually associate with a specific test condition. It is common to find published data measured only at one frequency (1 MHz typical). This article plots the changes in the impedance, attenuation, and phase as functions of frequency, twist pitch, load mismatch, material make-up, and dimensional tolerance of the twisted pair. With accurate calculations for the distributed parameters, you can build an accurate model for the twisted pair that reflects real transmission-line behavior for a range of frequencies.

References:

1. "Reference Data for Radio Engineers," Howard Sams & Co, 1956.
2. Lefferson, Peter, "Twisted Magnet Wire Transmission Line," 1971.
3. Magnuson, Phillip, "Transmission Line & Wave Propagation," 1981.
4. Brown et al, "Lines, Waves, and Antennas," 1973.

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